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A COMPARISON OF THREE DIFFUSION MODELS OF THE UPPER MIXED LAYER--ETC(U)

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NAVAL RESEARCH LABORATORY
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A Comparison of Three Diffusion Models of the Upper Mixed Layer of the Ocean

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The Munk-Anderson, Vager-Zilitinkevich, and Mellor-Yamada turbulent diffusion models were used to model the upper mixed layer of the ocean and were compared to determine differences in predicted mixed layer depths for simple cases of mixed layer deepening. The mixed layer depths predicted by the models were found to be very strongly dependent upon the value of the Richardson number above which the generation of turbulence by the mean velocity shear was stabilized by the stratification. The larger this critical value of the Richardson number, the further the mixed layer deepened before the stratification at the base of the mixed layer was able to suppress the turbulent mixing and stop further erosion of the thermocline.

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A COMPARISON OF THREE DIFFUSION MODELS OF THE UPPER MIXED LAYER OF THE OCEAN

I. INTRODUCTION

In recent years there has been increasing interest in modeling the upper mixed layer of the ocean. This is the turbulent region near the surface which is strongly mixed by the wind. The turbulence is generated either by wave action, by shear instability in the wind driven surface Ekman layer, or by convective instability.

The dynamics of the mixed layer is important because it affects the sea surface temperature, the storage of heat in the upper ocean, and the transfer of heat from the ocean to the atmosphere. The atmosphere obtains most of its energy from latent and sensible heat exchange with the ocean, and these fluxes of heat are strongly dependent upon the air-sea temperature difference. The sea surface temperature in turn depends upon the sea surface heat flux, the depth of the mixed layer, and the rate at which the mixed layer depth is changing. It is also influenced by the history of past warmings and deepenings which determine the temperature structure below the mixed layer down to the base of the seasonal thermocline. Therefore, prediction of the storage of heat in the upper ocean, the sea surface temperature, and the air-sea heat flux requires careful modeling of the mixing processes in the upper layers of the sea.

Note: Manuscript submitted October 18, 1976.

This work consists of a description and comparison of the eddy diffusion models of Munk and Anderson (1948), Vager and Zilitinkevich (1968), and Mellor and Yamada (Level 2 model, 1974). These models are parameterizations of the vertical fluxes of heat and momentum in a turbulent, stratified boundary layer. Such a parameterization is required to close and complete the equations for the conservation of heat and momentum for a model of the upper ocean.

The three models were chosen for comparison because they are fairly simple, easily implemented turbulence parameterizations. The Munk-Anderson model was derived in an empirical fashion from observations of stratified turbulence. The Vager-Zilitinkevich and Mellor-Yamada models are more recent attempts to develop simple turbulence models that take greater advantage of turbulence closure theory. The models were compared to determine how greatly their predictions of the depth of the mixed layer differ for identical conditions, and some attempt is made to relate the differences in predicted mixed layer depths to the structural differences of the models.

The eddy diffusion models were studied with a one-dimensional model in order to concentrate solely on the vertical processes occurring in the upper ocean. Therefore, only vertical diffusion is included in the equations and horizontal advection and diffusion are ignored. Vertical advection was also ignored on the assumption that the vertical upwelling typical of the open ocean, which could have been included in the model, does not play an important role in the dynamics of the mixed layer over short time periods (days).

Two problems of mixed layer deepening were examined with each of the three diffusion models. The first problem is that of the mixed layer depth achieved in an initially motionless, linearly stratified upper ocean due to the imposition of a constant wind stress and zero surface heat flux. The second problem is that of the mixed layer depth achieved in an initially motionless, weakly stratified upper ocean due to the imposition of a constant wind stress and a constant, positive heat flux at the sea surface. (A positive surface heat flux implies a flux of heat into the sea.) These two problems are representative of two distinct types of mixed layer deepening that might be expected in the real ocean. Denman (1973) refers to the first problem as a case of wind dominated deepening of the mixed layer and to the second as a case of heat dominated deepening.

Wind dominated deepening is characterized by a small or negligible sea surface heat flux. The deepening of the mixed layer is determined by the magnitude of the surface wind stress and the stability of the stratification of the upper ocean. Turbulence generated by the wind stress erodes the stable stratification at the base of the mixed layer, and as this water is entrained into the mixed layer, the mixed layer deepens.

Mixed layer models such as those of Kraus and Turner (1967) and Denman (1973) are based on the premise that a certain fixed fraction of the kinetic energy being imparted to the ocean by the surface wind stress is used to do work on the upper ocean by increasing its potential energy through turbulent mixing. In the case of wind dominated mixing where

the surface heat flux is very small, the wind stress can only do work on the upper ocean by deepening the mixed layer. As the mixed layer deepens, the potential energy of the upper ocean is increased by the entrainment and upward transport of denser water by turbulence. Hence, for the case of wind dominated mixing, the Denman and Kraus and Turner models predict that the mixed layer will continue to deepen as long as the wind is blowing. Observations of the actual behavior of the ocean's mixed layer seem to indicate that this behavior is unrealistic.

The models to be compared here make no assumptions regarding the fraction of kinetic energy input by the wind that acts to increase the potential energy of the upper ocean. It will be seen that the Vager-Zilitinkevich and Mellor-Yamada models predict that, for wind dominated deepening due to the imposition of a constant wind stress, the deepening of the mixed layer virtually ceases within a couple of days. At this point, all the kinetic energy generated by the wind is lost to viscous dissipation and the potential energy of the upper ocean remains constant.

Heat dominated mixing is characterized by a positive heat flux at the ocean's surface. The depth of the mixed layer is determined primarily by the magnitude of the surface heat flux and the strength of the wind stress. If there is a positive heat flux at the sea surface, the wind stress can do work on the upper ocean by mixing this heat throughout the mixed layer as well as by deepening the mixed layer and mixing dense water upwards. Since continued deepening of the mixed layer is not necessary in order for the wind stress to do work on the upper ocean, the Denman and Kraus and Turner models as well as the three models compared here predict that a stationary mixed layer depth is established if

the wind stress and surface heat flux remain fairly constant in time.

Because of the simple nature of the equations and the boundary and initial conditions for both problems of mixed layer deepening, the problems are amenable to a similarity scaling which results in a set of equations involving only one free parameter. Hence, in both problems the fairly stationary mixed layer depths achieved after several days of integration are a function of a single nondimensional parameter for each of the models. This considerably simplifies the analysis of the problems and provides some insight, not only into how the mixed layer depth depends on the surface wind stress and heat flux, the stratification, and the local inertial frequency, but also into why the mixed layer depths predicted by the diffusion models differ. Mellor and Durbin (1974) performed such a scaling for the problem of the stationary mixed layer depth determined by a constant positive surface heat flux, and noted that the mixed layer depth appeared to be given by

$$h \sim \frac{1}{2} \frac{u_\tau^3}{\beta g Q} \qquad (1)$$

in the limit $\mu \to \infty$. Here, μ is the ratio of the turbulent Ekman layer length scale u_τ/f and the Monin-Obukoff length scale $u_\tau^3/\beta g Q$, u_τ is the friction velocity associated with the wind stress, β is the coefficient of thermal expansion of sea water, g is the gravitational constant, and Q is the surface heat flux. It should be noted that the mixed layer depth does not depend upon the Coriolis parameter f in this limiting case.

Models of the mixed layer fall into two general groups, integral models and grid point models. Integral models such as those of Kraus and

Turner (1967) and Denman (1973) treat the mixed layer as a vertically integrated layer and use the vertically averaged values of mixed layer quantities to predict the behavior of the mixed layer. Grid point models use finite differenced forms of the conservation equations over a spatially fixed grid to predict the vertical distributions of velocity and density with time. These models have much in common with the grid point atmospheric boundary layer models, and some models, such as those of Vager and Zilitinkevich (1968) and Mellor and Yamada (1975) have been used to model both atmospheric and oceanic boundary layers.

Integral models use depth averaged values for the density and velocity of the mixed layer, and this is justified on the grounds that turbulent mixing within the mixed layer rapidly mixes the layer to a fairly uniform composition. However, although the rate of mixing within the mixed layer is rapid, it is not instantaneous. For instance, if the depth of the mixed layer is about 30 meters and the effective turbulent diffusivity is about 100 cm^2/sec (Ostapoff and Worthem, 1972), the diffusive time scale within the mixed layer is about a day, and it will be seen that the diffusion models compared here predict that the mixed layer depth can take a couple of days or more to adjust to a change in the sea surface heat flux. The temperature and velocity structure within the mixed layer affects the response of the mixed layer to changes in the wind stress and heat flux at the surface.

Grid point models predict the vertical distribution of velocity and density almost continuously in space. Hence, they model the diffusion of heat and momentum within the mixed layer as well as the fluxes of heat and momentum at the surface and at the bottom of the mixed layer and

below. Another advantage of grid point models is that no assumptions need to be made concerning the behavior of the mixed layer itself. The turbulent diffusivity is determined by the mean velocity and density fields and the vertical distribution of the turbulence in turn defines the extent of the mixed layer.

The advantage of integral mixed layer models is their simplicity and speed of computation. The limitations of computation time in three-dimensional numerical simulations of the ocean may prohibit the use of mixed layer models requiring high vertical resolution. However, for one-dimensional simulations of the upper ocean where horizontal inhomogeneities are ignored, the computation time required for integrations of days or months with any but the most complicated grid point models is not substantial, and these models are to be preferred over integral models.

Increased understanding of the dynamics of the mixed layer through observation and simulation with grid point models may allow the optimization of integral mixed layer models for particular oceanic applications.

II. DESCRIPTION OF THE UPPER OCEAN

The temperature structure of the upper ocean has a major effect on the vertical turbulent diffusivity. The upper ocean is warmed by solar radiation, exchanges heat with the atmosphere through sensible heat exchange, and loses heat due to short wave back radiation and evaporation. During the spring and summer heating season, solar heating predominates and a stable density gradient is built up in the upper layers. Winds blowing over the sea generate turbulent mixing in the near surface water,

forming a layer of fairly uniform temperature and salinity. The mixing of the near surface water causes a large, stable temperature gradient, a thermocline, to develop between the wind mixed layer and the water below. This temperature gradient inhibits vertical mixing and tends to prevent turbulence from penetrating below the thermocline. Deepening of the mixed layer due to increasing winds or warming of the layer through heating at the surface act to strengthen the thermocline, creating an even stronger barrier to the penetration of heat and momentum into deeper water.

The effective vertical diffusivity in the upper ocean can vary from values on the order of 100 cm^2/sec in the turbulent, wind mixed surface layer to values approaching the molecular rates of diffusion of heat and momentum in nonturbulent regions below the thermocline.

III. DESCRIPTION OF THREE EDDY DIFFUSION MODELS

The first significant modeling of a diffusive boundary layer in the presence of rotation was by Ekman (1905). Ekman analysed the response of a homogeneous ocean with constant vertical diffusion coefficient K to a suddenly imposed surface wind stress τ. He found that because of the earth's rotation the frictional influence of the wind was limited to a boundary layer of depth $\pi\sqrt{2\,K/f}$, where f is the Coriolis parameter. This, the Ekman layer depth, is the depth at which the current has been reduced to $e^{-\pi}$ times its value at the surface. Ekman noted, however, that the rate of vertical diffusion in the real ocean would depend upon the stratification.

A. Munk-Anderson Diffusion Model

Munk and Anderson (1948) reasoned from dimensional arguments that the effective vertical diffusivity in a turbulent, stratified fluid should depend strongly upon the Richardson number Ri, where

$$Ri = \frac{g \frac{1}{\rho} \frac{\partial \rho}{\partial z}}{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2} \quad . \tag{2}$$

Here g is the acceleration of gravity, ρ is the potential density of the fluid, z is the vertical coordinate, and u and v are the horizontal velocity components. The Richardson number is a measure of the degree of balance between the stabilizing effect of the density gradient and the destabilizing effect of the mean velocity shear, and is an indicator of the degree to which the turbulent mixing is being suppressed by the density stratification.

Munk and Anderson postulated the functional forms

$$K_M = K_o(1 + \beta_M Ri)^{-n_M}$$

and (3)

$$K_H = K_o(1 + \beta_H Ri)^{-n_H}$$

for the vertical diffusion coefficients for momentum and heat K_M and K_H. Here K_o is the magnitude of the vertical diffusion coefficients in the limit $Ri \to 0$ and n_M, n_H, β_M, and β_H are constants. From an analysis of wind profile measurements in the atmospheric boundary layer, Munk and Anderson determined that these constants should be assigned the values

$$n_M = \frac{1}{2}, \quad n_H = \frac{3}{2}, \quad \beta_M = 10, \quad \beta_H = \frac{10}{3} .$$

B. Vager-Zilitinkevich Diffusion Model

Vager and Zilitinkevich (1968) developed a diffusion model based upon the assumption that the rate of vertical diffusion of heat and momentum is proportional to the square root of the kinetic energy of the turbulent eddies B and to the length scale of the turbulence ℓ, so that

$$K_M = K_H = c_o \ell \sqrt{B} , \tag{4}$$

where c_o is a constant. In their model, the kinetic energy of the turbulence is predicted by a simplified form of the equation for the turbulent kinetic energy

$$\frac{\partial B}{\partial t} = \alpha_B \frac{\partial}{\partial z}\left(K_M \frac{\partial B}{\partial z}\right) + K_M\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] - K_H \beta g \frac{\partial T}{\partial z} - c\,\frac{B^2}{K_M} , \tag{5}$$

where c is a universal constant with value 0.046, α_B is a constant with value 0.73, and β is the coefficient of thermal expansion of the fluid. The terms on the right side of Eq. (5) model, respectively, the diffusion and production of turbulent energy, the conversion of turbulent energy to potential energy, and the dissipation of turbulent energy. The constants c and c_o are related by $c_o = c^{1/4}$. In solving Eq. (5), the flux of turbulent kinetic energy at the ocean's surface is taken to be zero, so that

$$\alpha_B K_M \frac{\partial B}{\partial z} = 0 \quad \text{at } z = 0 . \tag{6}$$

The length scale ℓ is determined from the hypothesis of Zilitinkevich and Laykhtman concerning the length scale of turbulence in a turbulent boundary layer, in which

$$\ell = - k \frac{\sqrt{B}/\ell}{\frac{\partial}{\partial z} \frac{\sqrt{B}}{\ell}} , \tag{7}$$

where k is von Karman's constant, taken to be .4. This expression can be integrated to the more convenient form

$$\ell = k\sqrt{B} \left\{ \frac{z_o}{\sqrt{B_o}} + \int_{z_o}^{z} \frac{dz}{\sqrt{B}} \right\} , \tag{8}$$

where z_o is the roughness parameter typical of the boundary and B_o is the turbulent kinetic energy at the boundary. Combining Eqs. (4) and (8), the vertical diffusion coefficients can be expressed as

$$K_M = K_H = kc_oB \left\{ \frac{z_o}{\sqrt{B_o}} + \int_{z_o}^{z} \frac{dz}{\sqrt{B}} \right\} . \tag{9}$$

C. Mellor-Yamada Level 2 Diffusion Model

Mellor and Yamada (1974) have developed a series of four turbulence closure models for planetary boundary layers labeled Level 1 to Level 4 in order of increasing complexity. The models were developed through systematic scaling of the equations for the turbulent fluxes of heat and momentum. The modeling of the triple correlations and dissipation terms in these equations was based on hypotheses proposed by Rotta

and Kolmogoroff and the values of constants used in such terms were determined from unstratified turbulence data.

Because of its simple form, the Level 2 model was chosen for comparison with the Munk-Anderson and Vager-Zilitinkevich models. For the Level 2 model, the turbulence fluxes of momentum and heat can be expressed in eddy coefficient form where the eddy coefficients depend upon the local mean velocity and density gradients. Henceforth, reference to the Mellor-Yamada model will refer to their Level 2 turbulence closure model. Mellor and Durbin (1975) have applied this model to the mixed layer of the ocean.

The vertical diffusion coefficients for the Level 2 model are defined as

$$K_M = \ell \sqrt{2B}\, S_M ,$$

and

$$K_H = \ell \sqrt{2B}\, S_H , \tag{10}$$

where the stability functions S_M and S_H are given by

$$S_H = 0.53664 - 1.97808\, R_f/(1 - R_f) ,$$

and

$$S_M = S_H \frac{(0.52 - 1.404\, R_f/(1 - R_f))}{(0.688 - 2.068\, R_f/(1 - R_f))} . \tag{11}$$

The flux Richardson number R_f is related to the Richardson number (2) by

$$R_f = \frac{S_H}{S_M} Ri \; . \tag{12}$$

The turbulent energy B is calculated from a highly simplified turbulent energy equation given by

$$0 = K_M \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right] - K_H \beta g \frac{\partial T}{\partial z} - \frac{(2\,B)^{3/2}}{15 \ell} \; . \tag{13}$$

Equation (13) expresses a balance among the production of turbulent energy from the mean velocity shear, the conversion of turbulent energy to potential energy, and the dissipation of turbulent energy. The turbulence length scale ℓ is determined from the vertical extent of the turbulence field as

$$\ell = 0.10 \frac{\int_{-\infty}^{0} z \sqrt{B}\, dz}{\int_{-\infty}^{0} \sqrt{B}\, dz} \; . \tag{14}$$

The differences in the three diffusion models can be seen most readily by considering how the turbulent diffusion coefficients depend upon the Richardson number.

For the Munk-Anderson model,

$$K_M = K_{M_o} (1 + 10\, Ri)^{-1/2}$$

and

$$K_H = K_{H_o} \left(1 + \frac{10}{3} Ri \right)^{-3/2} \; . \tag{15}$$

Hence,

$$K_M/K_{M_o} = (1 + 10\,Ri)^{-1/2}$$

and (16)

$$K_H/K_{H_o} = \left(1 + \frac{10}{3}\,Ri\right)^{-3/2} ,$$

where K_{M_o} and K_{H_o} are the diffusion coefficients for an unstratified ocean where Ri is zero.

For the Vager-Zilitinkevich model

$$K_M = K_H \simeq \ell^2 \,|\bar{v}_z|\,(1 - Ri)^{1/2} , \tag{17}$$

where

$$|\bar{v}_z| = \left(\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right)^{1/2} . \tag{18}$$

Hence ,

$$K_M/K_{M_o} = K_H/K_{H_o} \simeq (1 - Ri)^{1/2} . \tag{19}$$

For the purpose of deriving Eq. (17) the terms representing the storage and diffusion of turbulent kinetic energy in Eq. (5) were ignored. In actual integrations using the Vager-Zilitinkevich model, these terms were not found to affect predicted mixed layer depths significantly.

For the Mellor Level 2 model

$$K_M = \sqrt{15}\,\left(S_M\right)^{3/2}\ell^2\,|\bar{v}_z|\,\left(1 - \frac{S_H}{S_M}\,Ri\right)^{1/2} , \tag{20}$$

and

$$K_H = \sqrt{15}\left(S_M\right)^{1/2} S_H \ell^2 |\bar{v}_z| \left(1 - \frac{S_H}{S_M} Ri\right)^{1/2} . \quad (20)$$

Hence

$$K_M/K_{M_o} = \left(S_M/S_{M_o}\right)^{3/2} \left(1 - \frac{S_H}{S_M} Ri\right)^{1/2} ,$$

and (21)

$$K_H/K_{H_o} = \left(S_M/S_{M_o}\right)^{1/2} \left(S_H/S_{H_o}\right) \left(1 - \frac{S_H}{S_M} Ri\right)^{1/2} .$$

Equations (16), (19), and (21) are plotted in Fig. 1. The critical Richardson number Ri_c for which turbulent mixing ceases differs significantly for each of the three models. The Vager-Zilitinkevich and Mellor-Yamada models predict that turbulence will be "shut off" for Richardson numbers exceeding 1.0 and .23 respectively. In contrast, the Munk-Anderson model predicts a very gradual decrease in the intensity of the turbulent mixing as the Richardson number increases.

IV. ADJUSTMENT OF THE DIFFUSION MODELS

For a given set of boundary and initial conditions it was felt that the three diffusion models should predict vertical diffusivities within the mixed layer of about the same magnitude as the vertical diffusivities that have been observed in the upper ocean under similar conditions. Bowden, Howe, and Tait (1970) derived estimates of the diffusivity of heat K_H of about 93 cm^2/sec in the near surface layer from diurnal temperature variations, and Ostapoff and Worthem (1972) deduced an

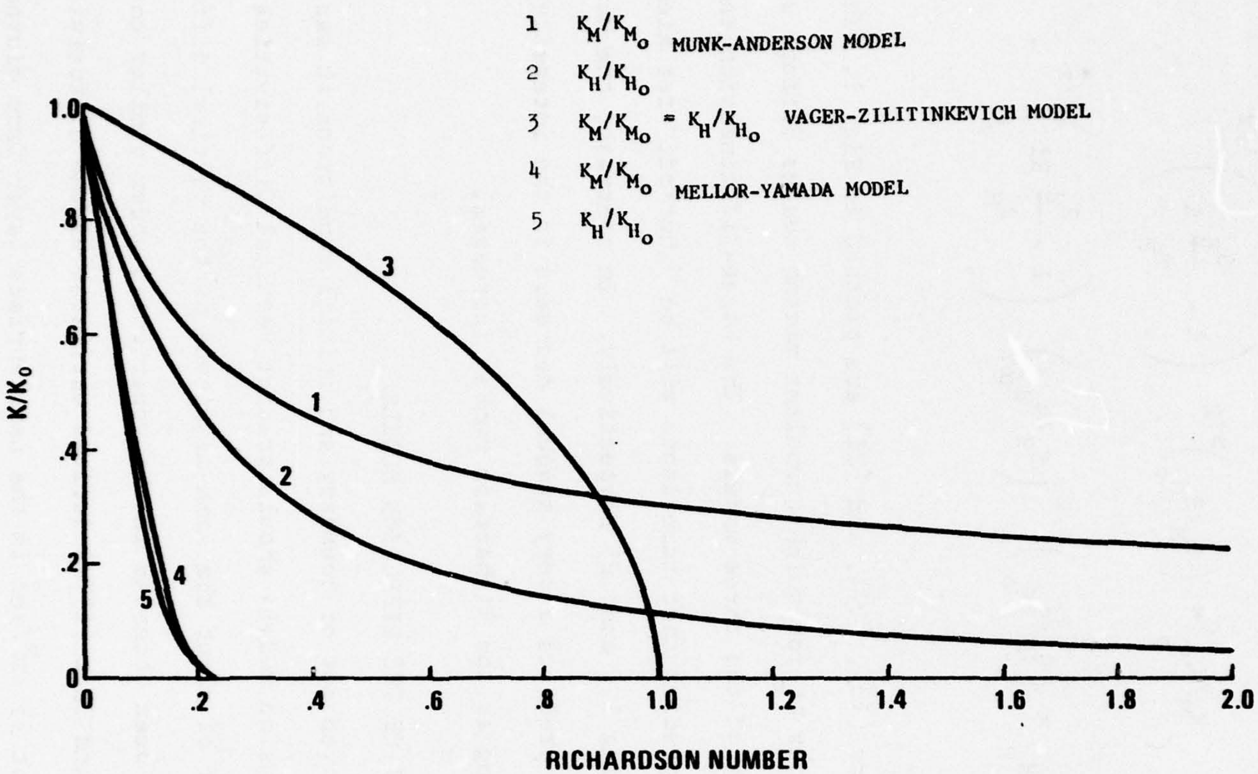

Fig. 1 — The approximate dependence of K/K_o on the Richardson number, where K/K_o is the ratio of the stratified and neutral values of the turbulent diffusion coefficients, shown for both momentum (subscript M) and heat (subscript H) for the three diffusion models

average value for K_H of about 72 cm^2/sec in the mixed layer during a day when the water was being stably stratified by solar heating and the wind speed was in the range of 6 to 7 m/sec. Since the conditions under which these observations were made were similar to the conditions imposed on the diffusion models for the two problems of mixed layer deepening, where the wind-stress was one dyne and the mixed layer depth was fairly shallow, about 30 meters, the diffusion models were adjusted to predict a maximum diffusivity of about 100 cm^2/sec.

Munk and Anderson (1948) proposed that the parameter K_o in their model (Eq. 2) should be a function only of the wind stress, and their paper gives a value for K_o of 150 cm^2/sec for a wind stress of one dyne. In the upper ocean, it might be expected that K_o should depend upon the depth of the mixed layer as well as the wind stress since as the mixed layer deepens, the length scale of the turbulent eddies and the magnitude of the vertical diffusivity would be expected to increase. Hence, for a given wind stress, a particular value for K_o might be too small for a very deep mixed layer and too large for a very shallow mixed layer. For the two problems of mixed layer deepening studied here, K_o was set equal to 100 cm^2/sec.

The Vager-Zilitinkevich model as described in Section 3 predicted vertical diffusivities greater than 300 cm^2/sec for the problems studied which seemed too large, so the turbulence length scale as predicted by Eq. (7) was reduced by a factor of three. The Mellor-Yamada model predicted diffusivities in fair agreement with the observed values and no adjustment to it was made.

It must be noted that the adjustments to the magnitudes of the eddy diffusivities, which did not affect the functional dependence of the diffusivities upon the stratification, did not significantly change the mixed layer depths predicted by the diffusion models for the two mixed layer deepening problems studied.

Well below the mixed layer the diffusion models predict no turbulent mixing, and thus the molecular diffusivities assert themselves. Although observations indicate that intermittent turbulent mixing enhances vertical diffusion below the mixed layer (Grant, Moillet, and Vogel, 1968), mechanisms for the generation of turbulence in this region (such as the breaking of internal waves) were ignored here. The mixed layer depths predicted by the models were not found to be sensitive to the eddy diffusivity imposed below the mixed layer provided it was small relative to the rate of mixing within the mixed layer.

V. ONE-DIMENSIONAL UPPER OCEAN MODEL EQUATIONS

The equations for the conservation of heat and momentum for a one-dimensional model of the upper ocean where the only spatial variations are with respect to the vertical coordinate z can be written as

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial z}\left(K_M \frac{\partial u}{\partial z}\right) + fv ,$$

$$\frac{\partial v}{\partial t} = \frac{\partial}{\partial z}\left(K_M \frac{\partial v}{\partial z}\right) - fu , \qquad (22)$$

and

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(K_H \frac{\partial T}{\partial z}\right) .$$

A small upwelling velocity typical of the open ocean could have been included but was not considered to be important to the dynamics of the mixed layer on the time scale of the problems considered here. The salinity of the upper ocean was assumed to be uniform and the density of the seawater was taken to be linearly dependent upon the temperature, the coefficient of thermal expansion of seawater β being 2.3×10^{-4} $^{o}C^{-1}$.

The boundary conditions for Eqs. (22) can be written as

$$K_M \frac{\partial u}{\partial z} (z = 0) = u_\tau^2 ,$$

$$K_M \frac{\partial v}{\partial z} (z = 0) = 0 , \qquad (23)$$

and

$$K_H \frac{\partial T}{\partial z} (z = 0) = Q ,$$

where $u_\tau = (\tau/\rho)^{1/2}$ is the friction velocity associated with the wind stress τ, and Q is the sea surface heat flux.

For the first problem, the case of a constant imposed wind stress, zero surface heat flux, and a linear initial stratification, u_τ was taken to be 1 cm/sec and Q was taken to be zero, and at $t = 0$ the ocean was assumed to be at rest and stratified with a linear temperature gradient of .1 ^{o}C/meter.

For the second problem, the case of a constant imposed wind stress and a constant positive surface heat flux, u_τ was taken to be 1 cm/sec and Q was taken to be 3.55×10^{-3} ^{o}C-cm/sec which corresponds

to a surface heat flux of 300 cal/cm^2/day into the ocean. At t = 0 the ocean was taken to be at rest and unstratified.

VI. SCHEME FOR NUMERICAL SOLUTION OF THE OCEAN MODEL EQUATIONS

The Eqs. (22) for the mean velocity and temperature fields were solved numerically by finite differencing in space and time. A 2-level time differencing scheme was used. The diffusion terms were evaluated at the n + 1 time level for stability and the Coriolis terms were split equally between the n and n + 1 time levels. The diffusion coefficients were evaluated at time level n.

A uniform spatial grid was used with a spacing of two meters between grid points. The temperature, density, and velocity were specified at each grid point and the eddy coefficients were calculated at positions between the grid points. A time step of six minutes was used for the integrations.

VII. RESULTS OF NUMERICAL INTEGRATION OF THE DIFFUSION MODELS

A. Wind Dominated Deepening of the Mixed Layer

Figures 2 to 6 summarize the results of the integration of the three diffusion models for the first problem, the case of a constant wind stress of 1 dyne/cm^2, zero surface heat flux, and an initial stratification of .1 $^{\circ}$C/meter.

The deepening of the mixed layer with time is shown in Fig. 2. For the Vager-Zilitinkevich and Mellor-Yamada models, the mixed layer is fairly well defined, since the production of turbulent energy by the mean shear stops abruptly when the Richardson number exceeds its critical value Ri_c. For the Munk-Anderson model, the turbulent mixing is never

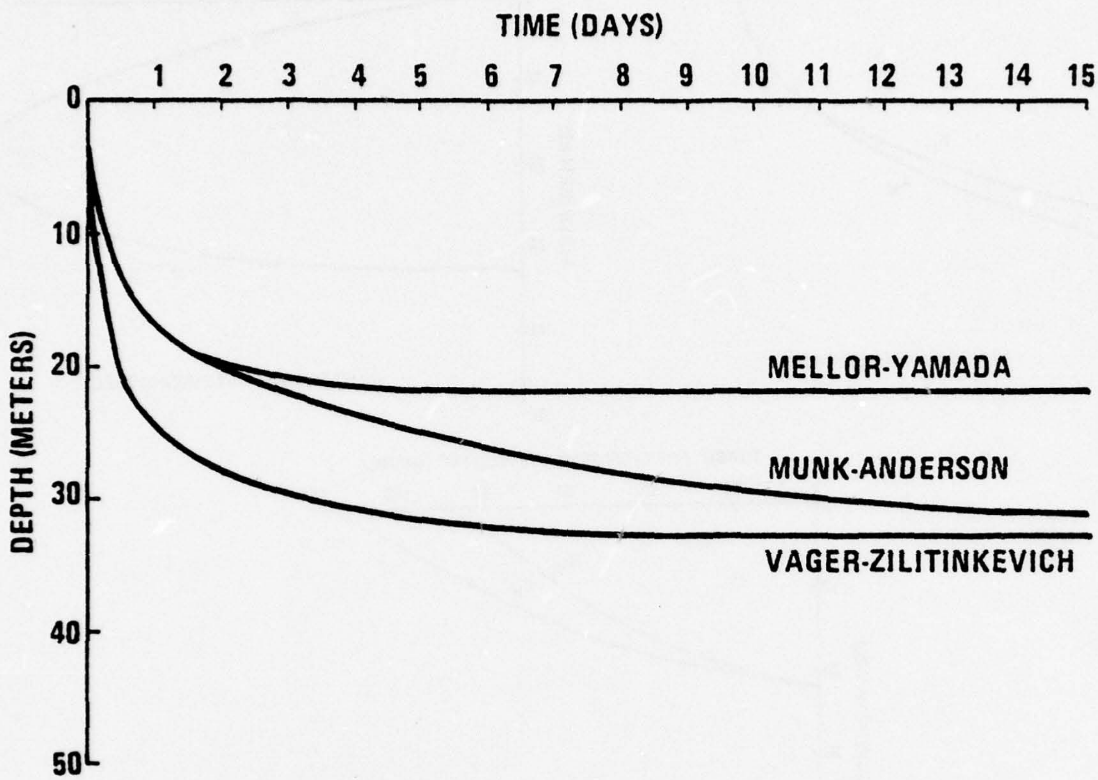

Fig. 2 — The mixed layer depth predicted by the three diffusion models for the problem of wind dominated deepening. The initial stratification was 0.1 °C/meter. The imposed wind stress was 1 dyne/cm^2 and the surface heat flux was set to zero.

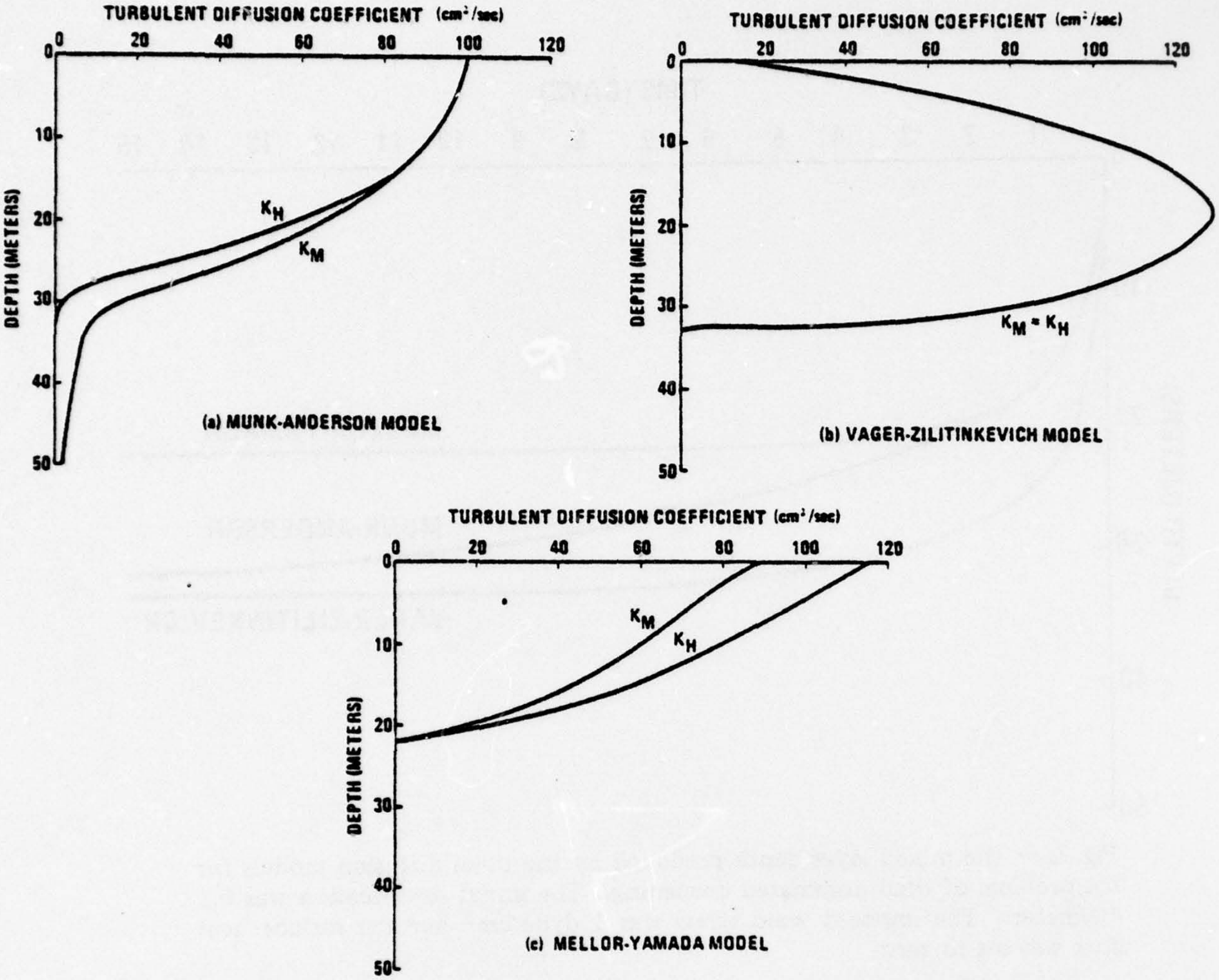

Fig. 3 — The profiles of the vertical diffusion coefficients K_M and K_H at $t = 15$ days

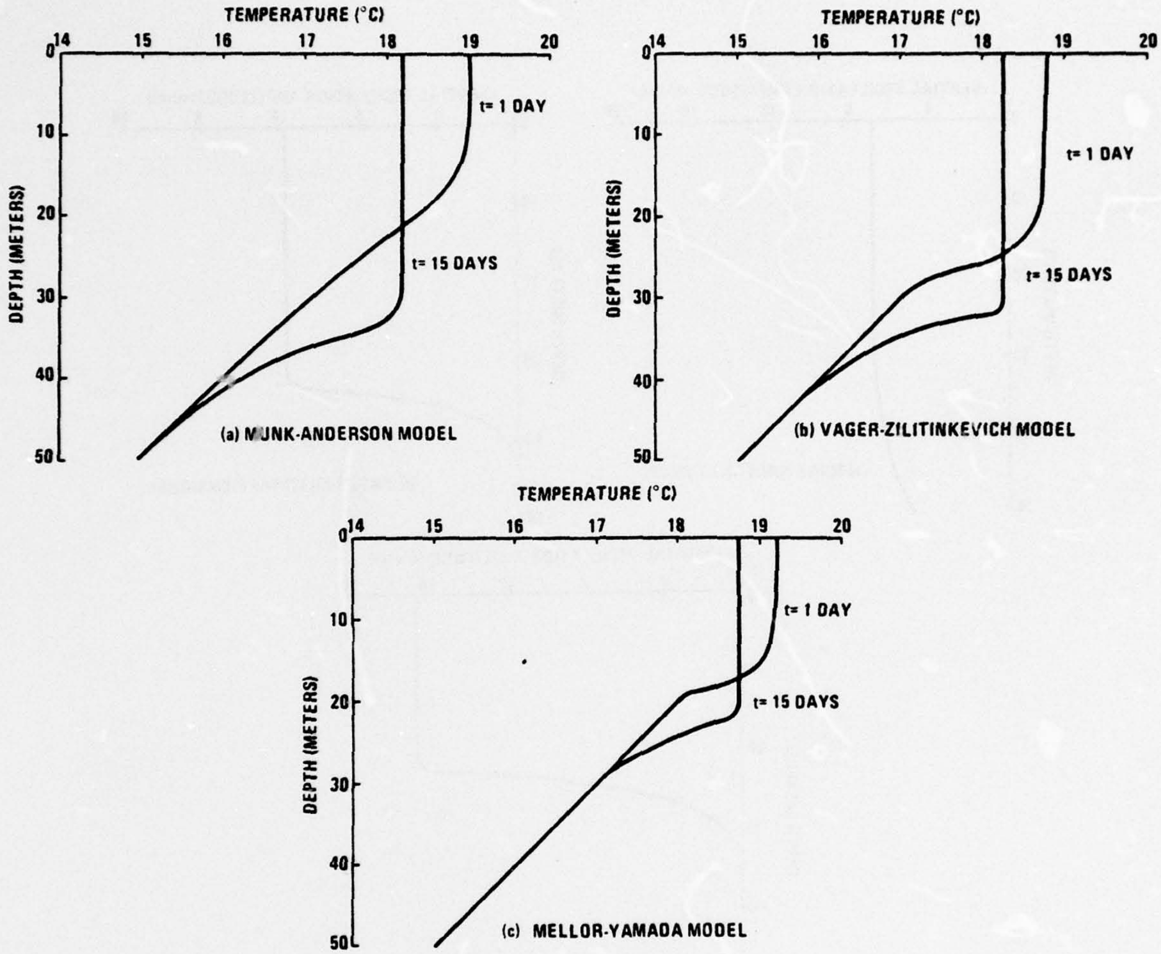

Fig. 4 — Temperature profiles at t = 1 day and t = 15 days after turning on the wind stress

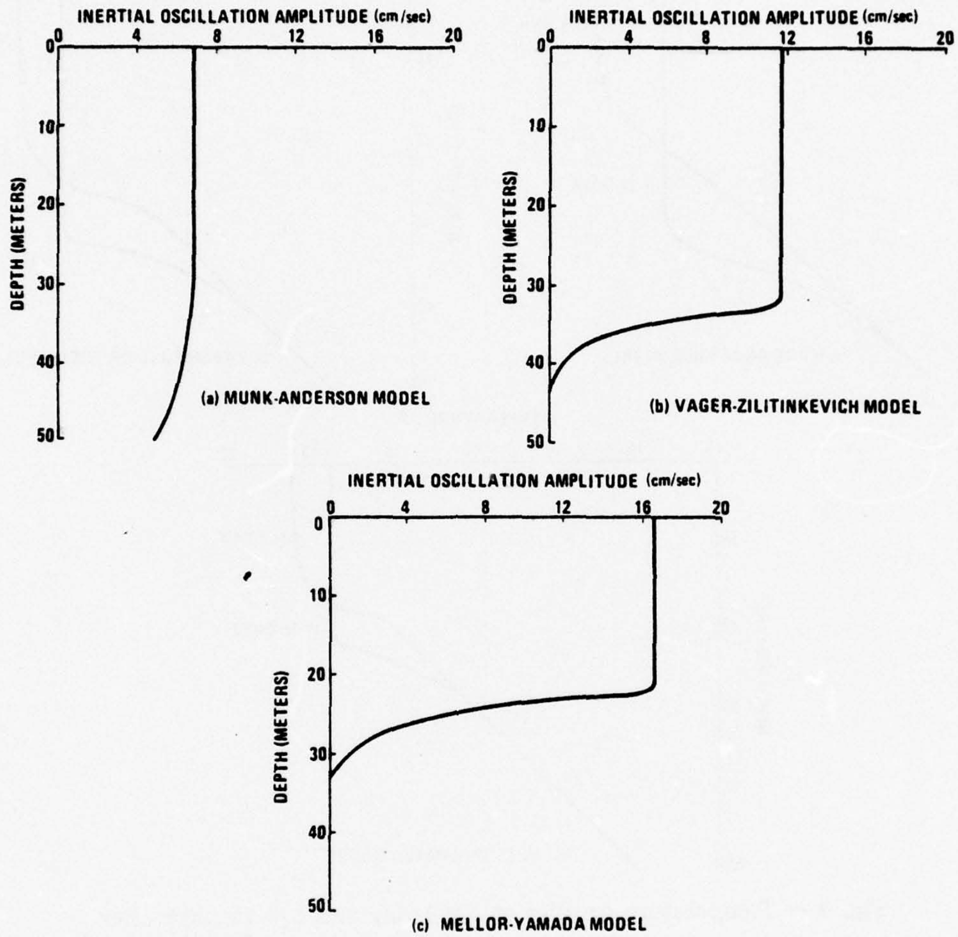

Fig. 5 — The inertial oscillation amplitude at t = 15 days after turning on the wind stress

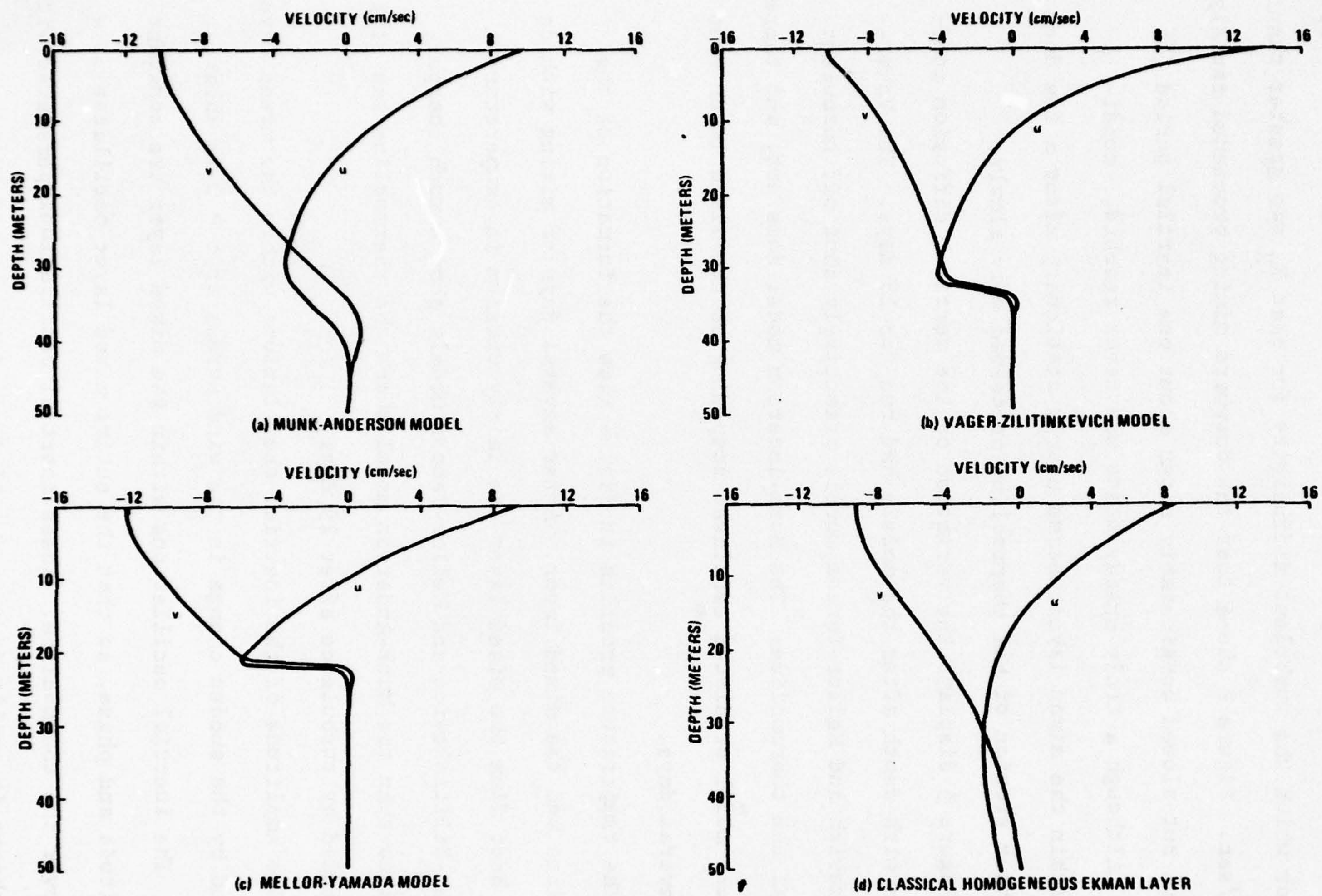

Fig. 6 — The mean velocity profiles in the mixed layer at t = 15 days after turning on the wind stress

completely shut off by the stratification. Hence, for the purpose of constructing Fig. 2, the mixed layer was taken to be the region of the upper ocean for which the turbulent diffusivity for heat K_H was greater than 1.0 cm^2/sec. Figure 2 shows that the downward mixing proceeded rapidly at first, but slowed considerably after about one inertial period (36 hours). Although a truly steady state was never reached, conditions within the mixed layer became nearly stationary after a few days and further erosion of the thermocline proceeded very slowly.

Figure 3 displays the variation of the vertical diffusion coefficients with depth after the models had run for 15 days. The Vager-Zilitinkevich and Mellor-Yamada models effectively shut off turbulent mixing at the thermocline. The Munk-Anderson model does not, and therefore their model predicted continued deepening of the mixed layer even after several days.

The temperature profiles in Fig. 4 show the formation of the thermocline and the mixed layer. After several days of mixing with no surface heat flux the mixed layer was fairly uniform in temperature. The Vager-Zilitinkevich and Mellor-Yamada models show a much sharper thermocline than the Munk-Anderson model where the thermocline was still being eroded by turbulence after 15 days.

The amplitude of the inertial oscillations within the mixed layer generated by the sudden change in the wind stress at $t = 0$ is shown in Fig. 5. The inertial oscillations within the mixed layer are constant in amplitude and phase, so that the entire mixed layer oscillates as a 'slab' over the thermocline. These inertial oscillations persisted with little change in amplitude over the 15 days of integration because of

the very small (and probably unrealistic) damping due to the small value of K_M below the mixed layer. The amplitude of inertial oscillations in the upper ocean, besides being dependent upon changes in the direction and strength of the wind (Pollard and Millard, 1970), is roughly inversely proportional to the mixed layer depth and this relationship can be seen by comparing the mixed layer depths and inertial oscillation amplitudes predicted by the three models.

The mean velocity profiles are shown in Fig. 6. These were the mean velocities at each depth about which the inertial oscillations occurred. A velocity profile for a classical, steady, homogeneous Ekman layer with $K_M = 100$ cm^2/sec is included in Fig. 6 for comparison. An analysis of Eqs.(22) shows that the Ekman layer transport depends only on the wind stress and the Coriolis parameter and is directed 90° to the right of the wind stress. Therefore, all the models yield the same transport as that predicted by Ekman's (1905) theory, in which

$$\int_{-\infty}^{0} u dz = 0 ,$$

and (24)

$$\int_{-\infty}^{0} v dz = - \frac{u_\tau^2}{f} .$$

B. <u>Heat Dominated Deepening of the Mixed Layer</u>

Figure 7 shows the change of the mixed layer depth with time predicted by the three diffusion models for the second problem, the case of an imposed wind stress of 1 dyne/cm^2, an imposed surface heat flux

of 300 cal/cm^2/day, and negligible initial stratification. The qualitative behavior predicted by the three models was similar. The mixed layer mixed very deeply at first because of the absence of stratification, but as the stratification of the upper ocean increased due to the downward mixing of the heat entering at the surface, the deepening slowed and stopped. As the stratification continued to increase, the turbulent mixing was weakened still further, and the mixed layer depth decreased until a certain balance was reached between the production of turbulent energy by the mean shear and the production of potential energy by the turbulent mixing. At this point the mixed layer depth stabilized since the density gradient was just sufficient to stabilize the mean velocity shear at the base of the mixed layer.

Because of the larger value of the critical Richardson number of the Vager-Zilitinkevich model, this model predicted a deeper mixed layer than the Mellor-Yamada model for both problems of mixed layer deepening. A larger value of the critical Richardson number implies that a stronger density gradient is required to stabilize a given velocity shear. As the mixed layer deepens, the mean velocity shear near the base of the mixed layer decreases. Hence, the larger the value of the critical Richardson number, the deeper the mixed layer can penetrate before the turbulent mixing at the base of the mixed layer becomes suppressed by the stratification.

Figure 7 shows that the Munk-Anderson model took a longer time to establish a steady mixed layer depth for the second problem of mixed layer deepening than the two other diffusion models. The mixed layer depth for the Munk-Anderson model was not very well defined until about

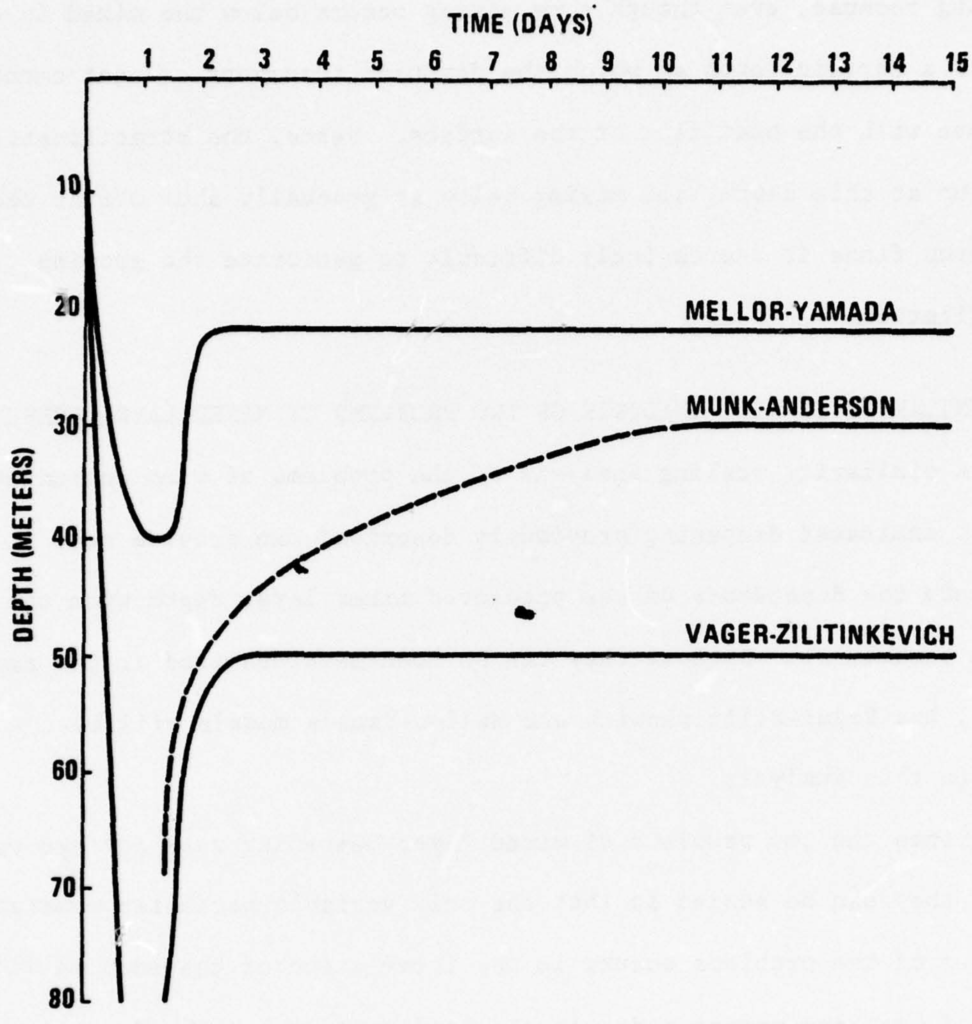

Fig. 7 — The mixed layer depth predicted by the three diffusion models for the problem of heat dominated mixing. The upper ocean was initially unstratified. The imposed wind stress was 1 dyne/cm^2 and the imposed sea surface heat flux was 300 cal/cm^2/day.

the tenth day of integration, and so the mixed layer depth up to that time is shown with a dotted line. A steady mixed layer depth becomes established with the Munk-Anderson model for problems of heat dominated deepening because, even though some mixing occurs below the mixed layer, there is a certain depth at which the downward transport of heat cannot keep pace with the heat flux at the surface. Hence, the stratification builds up at this depth, and mixing below is gradually shut off as the turbulence finds it increasingly difficult to penetrate the growing stratification.

VIII. SIMILARITY SCALING ANALYSIS OF TWO PROBLEMS OF MIXED LAYER DEEPENING

A similarity scaling analysis of the problems of wind dominated and heat dominated deepening previously described can provide some insight into the dependence of the predicted mixed layer depth upon the problem parameters. Because they can be nondimensionalized in the same fashion, the Vager-Zilitinkevich and Mellor-Yamada models will be included in this analysis.

Since the two problems of mixed layer deepening were defined very simply, they can be scaled so that the only variable parameter remaining in either of the problems occurs in the formulation of the eddy coefficients of heat and momentum due to the scaling of the Richardson number. For the problem of wind dominated deepening with a constant wind stress, no surface heating, and a linear initial stratification, this parameter is

$$\phi_1' = \frac{N^2}{f^2}, \tag{25}$$

which is the square of the ratio of the Brunt-Vaisala frequency N and the local inertial frequency f. Here, $N = \sqrt{\beta g \Gamma}$, where Γ is the initial stratification of the upper ocean, β is the coefficient of thermal expansion of the seawater, and g is the gravitational constant. For the problem of heat dominated deepening with a constant wind stress, a constant positive heat flux Q, and negligible initial stratification, the variable parameter remaining after scaling is

$$\phi_2' = \frac{\beta g Q}{f u_\tau^2} , \tag{26}$$

which is the ratio of the turbulent Ekman layer length scale u_τ/f and the Monin-Obukov length scale $u_\tau^3/\beta g Q$, where u_τ is the friction velocity associated with the wind stress.

This scaling can be made more general if approximations are made to the Vager-Zilitinkevich and Mellor-Yamada diffusion models to allow two additional parameters to be brought into the scaling which are characteristic of the diffusion models themselves. One of these parameters is α, a constant to which the eddy coefficients are proportional and which can be adjusted to change the magnitude of the eddy coefficients without affecting the nature of the dependence of the eddy coefficients upon the stratification. The other parameter is Ri_c, which is the critical Richardson number for each of the models above which the generation of turbulence by the mean shear is completely suppressed by the stratification.

Although α and Ri_c are fixed constants for both the Vager-Zilitinkevich and Mellor-Yamada diffusion models, by treating them as having a degree of variability some assessment can be made of the

importance of these constants in determining the mixed layer depths predicted by the two models.

The approximations are made to the models in order to simplify the scaling and at the same time present the models in a form that makes their similarities and differences more apparent. Both the Vager-Zilitinkevich and Mellor-Yamada models can be fairly well approximated by the general expression

$$K_M = K_H \simeq \alpha \ell^2 \, |V_z| \, (1 - Ri/Ri_c)^{\gamma} , \tag{27}$$

where K_M and K_H are the eddy diffusivities for heat and momentum, ℓ is the turbulence length scale, and $|V_z|$ is the absolute value of the mean velocity shear. For the Vager-Zilitinkevich model $\gamma = 1/2$, $\alpha = 1.0$, and $Ri_c = 1.0$, and for the Mellor-Yamada model $\gamma \simeq 3/2$, $\alpha \simeq 1.0$, and $Ri_c = 0.23$.

Putting the Vager-Zilitinkevich and Mellor-Yamada models into the form of Eq. (27) requires dropping the tendency and diffusion terms in the turbulent energy Eq. (5) for the Vager-Zilitinkevich model and simplifying the functions $S_M(Ri)$ and $S_H(Ri)$ of the Mellor-Yamada model Eq. (11) to the form $(1 - Ri/Ri_c)^{\gamma}$. The results of a test integration of the approximate form of the models differed little from the results of integration of the actual models. It will be mentioned here that all the results presented were obtained from integration of the diffusion models in their proper form as described in Sections III and IV.

For an unstratified flow where the Richardson number is zero, both the Vager-Zilitinkevich and Mellor-Yamada models essentually reduce to Prandtl's mixing length theory in which the eddy coefficients are

proportional to the square of the turbulence length scale and the mean velocity shear. For a stratified flow the eddy coefficients are modified by a function of the Richardson number which reduces the vertical diffusivity for a given velocity shear as the Richardson number increases, and hence, models the suppression of the turbulence by the stratification. For the approximate form of the Vager-Zilitinkevich and Mellor-Yamada models given by Eq. (27), this stratification function has the form

$$\left(1 - \frac{Ri}{Ri_c}\right)^{\gamma} . \qquad (28)$$

The values of the constants Ri_c and γ in Eq. (28) differentiate the Vager-Zilitinkevich and Mellor-Yamada models. The most significant difference between the models is that the critical Richardson number for the Vager-Zilitinkevich model $Ri_c = 1.0$ is much higher than that for the Mellor-Yamada model for which $Ri_c = .23$. A probably less significant difference is that the 'shape' of the stratification function (28) which is determined by the value of γ is quite different for the two models. The Vager-Zilitinkevich model with $\gamma = 1/2$ predicts considerably less suppression of the turbulent mixing for a given value of Ri/Ri_c than does the Mellor-Yamada model with $\gamma \simeq 3/2$. Since the parameter Ri_c is to be included in the scaling of the problem of mixed layer deepening, differences in the nondimensionalized mixed layer depth predicted by the models should be due to the difference in the 'shape' of the stratification function (28) represented by the different values of γ.

The scaling used to nondimensionalize the two problems of mixed layer deepening is similar to that suggested by Mellor and Durbin (1974).

A. Wind Dominated Deepening

For the case of wind dominated deepening with a constant wind stress, zero surface heat flux, and a linear initial temperature stratification Γ, the depth can be scaled as $u_\tau\sqrt{\alpha}/f$, the time as $1/f$, the velocity as $u_\tau/\sqrt{\alpha}$, the temperature as $\Gamma u_\tau\sqrt{\alpha}/f$, and the eddy coefficients as $u_\tau^2\sqrt{\alpha}/f$. After scaling, the only parameter remaining in the problem is

$$\phi_1 = \frac{N^2\alpha^2}{f^2 Ri_c} , \tag{29}$$

where $N = \sqrt{\beta g \Gamma}$ is the Brunt-Vaisala frequency determined by the initial stratification of the upper ocean. Therefore, the fairly steady mixed layer depth h achieved after several days of integration is a function of ϕ_1 and is given by

$$h = \frac{u_\tau\sqrt{\alpha}}{f} h^*(\phi_1) , \tag{30}$$

where h* is the dimensionless mixed layer depth.

Figure 8 shows the dependence of h* upon ϕ_1 found from numerical integration of the Mellor-Yamada and Vager-Zilitinkevich models. The two curves almost coincide, indicating that most of the difference in mixed layer depth predicted by the two models for the case of wind dominated deepening is due to the difference in Ri_c (which is taken into account by the scaling). The difference in the value of γ in the stratification function (28) of the two models and the diffusion of turbulent energy in the Vager-Zilitinkevich model did not significantly affect the

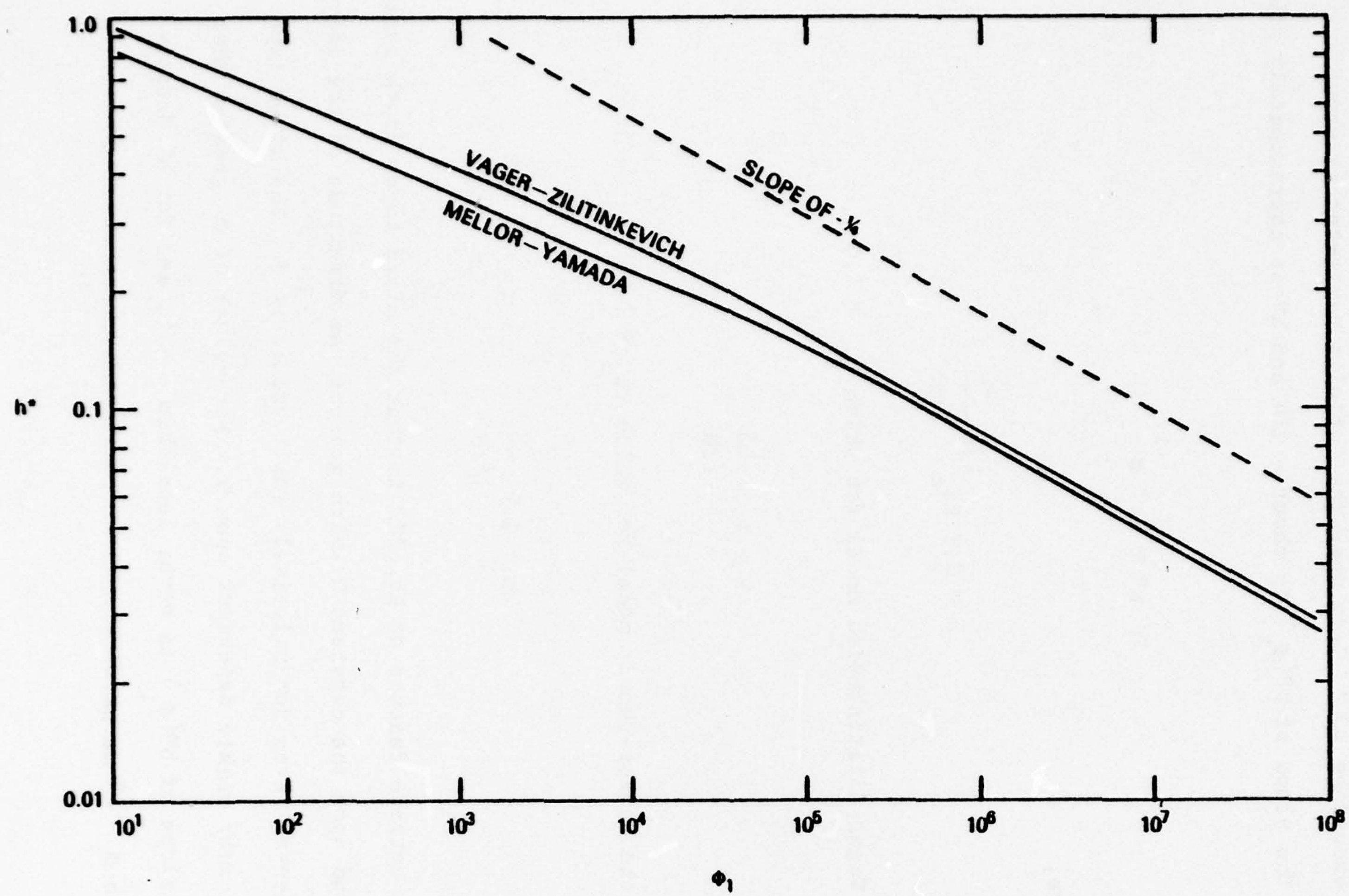

Fig. 8 — The dimensionless mixed layer depth h* versus ϕ_1 obtained by integration of the Vager-Zilitinkevich and Mellor-Yamada diffusion models for the case of constant wind stress and zero heating.

predicted mixed layer depths.

For values of ϕ_1 greater than 10^5 (which corresponds to stratifications exceeding 10^{-3} $^oC/cm$ when the Coriolis parameter is about 5×10^{-5} sec^{-1}), the slope of $h^*(\phi_1)$ is about $-1/4$ and h^* is approximately given by

$$h^* \simeq 2.7\ \phi^{-1/4} . \tag{31}$$

Therefore,

$$h \simeq 2.7\ Ri_c^{1/4} \frac{u_\tau}{\sqrt{fN}} . \tag{32}$$

For the Vager-Zilitinkevich model for which $Ri_c = 1.0$

$$h \simeq 2.7 \frac{u_\tau}{\sqrt{fN}} , \tag{33}$$

and for the Mellor-Yamada model for which $Ri_c = 0.23$

$$h \simeq 1.9 \frac{u_\tau}{\sqrt{fN}} . \tag{34}$$

A notable feature of Eq.(32) is that the mixed layer depth does not depend upon the constant α which governs the magnitude of the eddy coefficients. Even for relatively small values of ϕ_1, the mixed layer depth is only weakly dependent upon α. For values of ϕ_1 greater than 10, the slope of $h^*(\phi_1)$ is never less than $-1/6$, and for h^* proportional to $\phi_1^{-1/5}$ we have

$$h \propto (\alpha Ri_c)^{1/6} \frac{u_\tau}{f^{2/3} N^{1/3}} \quad . \tag{35}$$

The insensitivity of the mixed layer depth to the magnitude of the eddy coefficients was noted by Mellor and Durbin (1975) for their integrations of the Mellor-Yamada model.

Hence, for wind dominated deepening, the mixed layer depth is not very sensitive to either the magnitude of the eddy coefficients or the shape' of the stratification function (28). This is evidenced by the fact that the difference in the critical Richardson number accounts for most of the difference in the mixed layer depths predicted by the Vager-Zilitinkevich and Mellor-Yamada models.

From an integral analysis of the problem of wind dominated deepening of the mixed layer, Pollard, Rhines, and Thompson (1973) obtained

$$h = 1.7 \frac{u_\tau}{\sqrt{fN}} \quad , \tag{36}$$

which agrees fairly well with Eq. (34), the mixed layer depth predicted by the Mellor-Yamada model for values of the parameter ϕ_1 in the range 10^2 - 10^5. In deriving Eq. (36) Pollard, Rhines, and Thompson assumed a bulk Richardson number Ri_B for the mixed layer of one:

$$Ri_B = \frac{\beta g h (T - T_{-h})}{u^2 + v^2} = 1 \quad , \tag{37}$$

where T is the mean temperature and u and v are the mean horizontal velocity components of the mixed layer and T_{-h} is the temperature

immediately below the mixed layer. It is interesting that the integral analysis of Pollard, Rhines, and Thompson assuming a bulk Richardson number of 1.0 should give a result similar to that of the Mellor-Yamada model where the criterion for the deepening of the mixed layer is a local Richardson number of 0.23.

B. Heat Dominated Mixing

For the case of heat dominated deepening of the mixed layer with a constant wind stress, a constant positive surface heat flux Q, and negligible initial stratification, the nondimensionalization is similar to that for the previous problem except that the temperature must be scaled as $Q/u_T\sqrt{\alpha}$. After the problem has been scaled, the only variable parameter remaining is

$$\phi_2 = \frac{\beta g Q \alpha}{f u_T^2 Ri_c} . \tag{38}$$

Hence, the fairly steady mixed layer depth h achieved after several days of integration with a constant wind stress and surface heat flux is given by

$$h = \frac{u_T\sqrt{\alpha}}{f} h^*(\phi_2) . \tag{39}$$

Figure 9 shows the dependence of h* upon ϕ_2 obtained from numerical integration of the Vager-Zilitinkevich and Mellor-Yamada models.

A most immediately noticeable feature of Fig. 9 is that, although the graphs of $h^*(\phi_2)$ for the Vager-Zilitinkevich and Mellor-Yamada models have a similar shape, they do not coincide. The difference in mixed layer

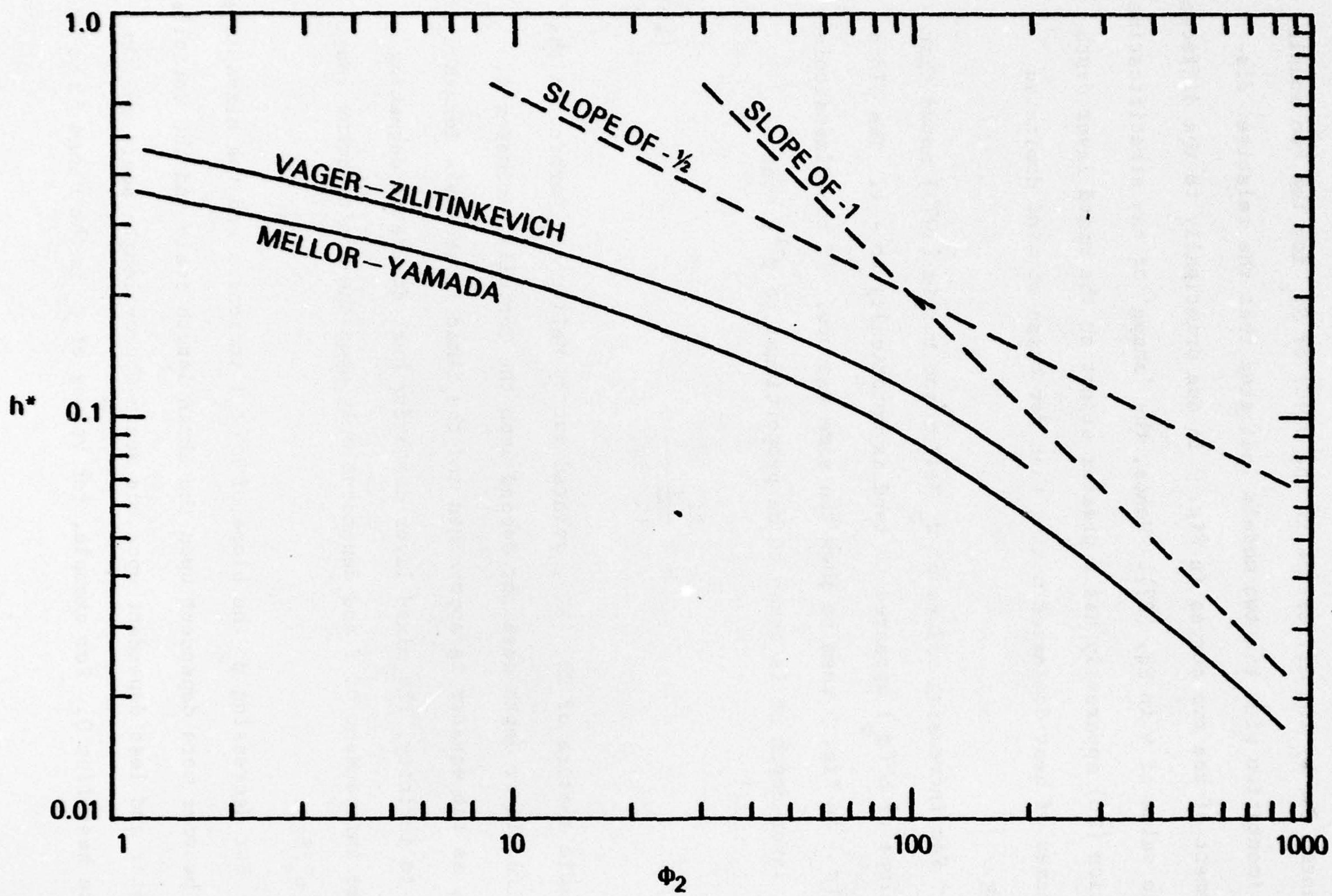

Fig. 9 — The dimensionless mixed layer depth h* versus ϕ_2 obtained by integration of the Vager-Zilitinkevich and Mellor-Yamada diffusion models for the case of constant wind stress and constant surface heat

depth predicted by the two models is greater than that which can be accounted for by the difference in the value of Ri_c for the two models. Experimentation with the two models indicated that the relative displacement of the two curves in Fig. 9 is due principally to the difference in the value of γ in Eq. (27). Hence, the 'shape' of the stratification function (28) apparently has a greater effect on the mixed layer depth for cases of heat dominated mixing than for cases of wind dominated mixing.

For increasing values of ϕ_2 Mellor and Durbin (1974) noted that the slope of $h^*(\phi_2)$ appeared to tend asymptotically to - 1. The plots of $h^*(\phi_2)$ in Fig. 9 seem to show the same feature. If the dimensionless mixed layer depth h^* is taken to be proportional to ϕ^{-1}, then

$$h \propto \frac{Ri_c}{\sqrt{\alpha}} \frac{u_\tau^3}{\beta g Q} . \tag{40}$$

A notable feature of Eq. (40), pointed out by Mellor and Durbin, is that the mixed layer depth does not depend upon the Coriolis parameter f. Hence, as the equator is approached and the Ekman layer scale length u_τ/f tends to infinity, the mixed layer depth for heat dominated deepening becomes independent of f and dependent only upon the Monin-Obukov length scale $u_\tau^3/\beta g Q$.

For decreasing ϕ_2 the slope of $h^*(\phi_2)$ increases and the mixed layer depth becomes more dependent upon the Ekman length scale and the Coriolis parameter and less dependent upon the Monin-Obukov length scale and the surface heat flux Q. For example, for values of ϕ_2 in the range 50 to 100

(which corresponds to a range of the surface heat flux of 200 to 400 cal/cm^2-day when the wind stress is about one dyne and the Coriolis parameter is about 5×10^{-5} sec^{-1}) the slope of $h^*(\phi_2)$ is about $-1/2$ and

$$h \propto Ri_c^{1/4} \frac{u_\tau^2}{\sqrt{f\beta g Q}} . \tag{41}$$

Equation (40) is essentially the same as the mixed layer depth dependence predicted by the Kraus and Turner model (1967) for heat dominated mixed layer deepening at all latitudes. The Kraus and Turner model assumes that turbulence within the mixed layer is generated by wave action rather than by the mean velocity shear within the surface Ekman layer, and therefore, the depth of the mixed layer is not influenced by the earth's rotation. Since the Kraus and Turner model assumes that the rate at which kinetic energy is imparted to the ocean by the wind (scaled by Kraus and Turner as u_τ^3) is proportional to the rate of increase of the potential energy of the upper ocean, which is scaled as $\beta g Q h$ when the mixed layer depth is fairly steady with time, we have

$$u_\tau^3 \propto \beta g Q h . \tag{42}$$

Hence, the mixed layer depth dependence of the Kraus and Turner model for heat dominated mixing is given by

$$h \propto \frac{u_\tau^3}{\beta g Q} . \tag{43}$$

Therefore, a significant difference between the Kraus and Turner mixed

layer model and the models of Vager and Zilitinkevich and Mellor and Yamada is that the mixed layer depths predicted by the latter are independent of the Coriolis parameter only in the limit as the ratio of the Ekman layer length scale and the Monin-Obukov length scale tends to infinity.

We note here that for an Ekman boundary layer the rate at which kinetic energy is being imparted to the ocean by the wind stress could be scaled as the product of the wind stress and the mean velocity of the Ekman layer, in other words, as u_τ^4/fh rather than as u_τ^3. Then, assuming the rate of addition of kinetic energy to the ocean to be proportional to the rate of change of the potential energy of the upper ocean, we obtain

$$h \propto \frac{u_\tau^2}{\sqrt{f\beta gQ}} . \tag{44}$$

However, such arguments appear to give valid results only for a small range of values of the parameter ϕ_2. Probably no simple theory of heat dominated deepening will yield results that agree with the results of integration of the Ekman layer Eqs. (22) over a wide range of values of the problem parameters.

IX. SUMMARY AND CONCLUSIONS

The general characteristics of the mixed layer predicted by the three diffusion models that were compared are similar. Turbulence generated throughout the mixed layer by the instability of the mean velocity shear keeps the layer mixed to a fairly uniform density and simultaneously

creates a sharp density gradient at the base of the mixed layer which suppresses turbulent mixing and inhibits the diffusion of heat and momentum into the region below.

The structure of the mean velocity field within the mixed layer consists primarily of an Ekman spiral which is modified by the stratification and maintained by the wind stress. Superimposed on the Ekman spiral are inertial oscillations generated by recent changes in the direction and strength of the wind. Because of the fairly high vertical diffusivity within the mixed layer and the large drop in the vertical diffusivity at the base of the mixed layer, inertial oscillations within the mixed layer tend to have fairly constant amplitude and phase and the whole mixed layer oscillates on top of the thermocline like a slab over a slippery surface.

For the case of wind dominated deepening of the mixed layer with a constant wind stress, no surface heat flux, and a linear initial stratification of the upper ocean, the three diffusion models predicted that the mixed layer would initially deepen very rapidly, but that the deepening would be slowed by the increasing stratification at the base of the mixed layer after about one inertial period. The Vager-Zilitinkevich and Mellor-Yamada models predicted that the deepening of the mixed layer would essentually stop after a couple of days, whereas the Munk-Anderson model predicted continued slow deepening of the mixed layer for the next several days. The difference in behavior of the models is probably due to the fact that, for the Vager-Zilitinkevich and Mellor-Yamada models the generation of turbulence by the mean shear is completely suppressed when the Richardson number exceeds a certain critical value Ri_c, whereas, for the

Munk-Anderson model the turbulent mixing is never completely shut off as the Richardson number increases and therefore, the turbulence is able to erode the thermocline at the base of the mixed layer even though the Richardson number in this region is fairly high.

A similarity scaling of the problem of wind dominated deepening of the mixed layer showed that almost all of the difference in mixed layer depth predicted by the Vager-Zilitinkevich and Mellor-Yamada models could be accounted for by the difference in the critical Richardson number of the two models. For values of the dimensionless parameter $N^2\alpha^2/f^2Ri_c$ corresponding to a thermal stratification exceeding 10^{-3} °C/cm and a value of the Coriolis parameter f of 5×10^{-5} sec^{-1}, the dependence of the mixed layer depth h upon the problem parameters for the Vager-Zilitinkevich and Mellor-Yamada models was found to be

$$h \simeq 2.7\, Ri_c^{1/4} \frac{u_\tau}{\sqrt{fN}}, \tag{32}$$

where N is the Brunt-Vaisala frequency corresponding to the initial stratification of the upper ocean, u_τ is the friction velocity associated with the wind stress, and α is a parameter governing the magnitude of the eddy coefficients. Since the values of the critical Richardson number Ri_c for the Vager-Zilitinkevich and Mellor-Yamada models are 1.0 and 0.23 respectively, the Vager-Zilitinkevich model predicted mixed layers about 30% deeper than those predicted by the Mellor-Yamada model for the cases of wind dominated deepening studied.

For wind dominated deepening, the mixed layer depth predicted by all three diffusion models was found to be somewhat insensitive to the

magnitude of the eddy coefficients. Hence, the parameterization of the turbulence length scale for the Vager-Zilitinkevich and Mellor-Yamada models or the determination of K_o, the vertical diffusivity for unstratified turbulent mixing for the Munk-Anderson model, is not as critical to the prediction of the mixed layer depth as the parameterization of the stabilization of the turbulent mixing by the stratification.

For the case of heat dominated deepening of the mixed layer with a constant wind stress, a constant positive heat flux at the surface, and negligible initial stratification of the upper ocean, the three diffusion models predicted similar behavior for the mixed layer. The mixed layer depth increased rapidly at first due to the absence of stratification, but then decreased suddenly after a couple of days as the stratification in the upper ocean increased, and stabilized at a depth such that the stratification at the base of the mixed layer was just sufficient to stabilize the mean velocity shear. The mixed layer then continued to warm due to the positive surface heat flux, whereas the region below the mixed layer, where the vertical diffusivity is relatively small, remained at a fairly constant temperature.

A similarity scaling of the problem of heat dominated mixing indicated that the difference in mixed layer depth predicted by the Vager-Zilitinkevich and Mellor-Yamada models was greater than that which could be accounted for by the difference in the critical Richardson number of the two models. If for the Vager-Zilitinkevich and Mellor-Yamada models the Richardson number is normalized by Ri_c, then the Mellor-Yamada model is found to predict less suppression of the turbulent mixing for a given value of Ri/Ri_c than the Vager-Zilitinkevich model. This difference in

the form of the function that governs the suppression of the turbulent mixing by the stratification was found to account for the additional difference in mixed layer depth predicted by the Vager-Zilitinkevich and Mellor-Yamada models for cases of heat dominated mixing. Hence, although it was thought that perhaps such models could be parameterized just in terms of the critical Richardson number characteristic of the particular model, it appears that this cannot be done.

The similarity scaling of the problem of heat dominated mixing also indicated that, as noted by Mellor and Durbin (1974), as the ratio of the Ekman layer length scale u_τ/f and the Monin-Obukov length scale $u_\tau^3/\beta gQ$ tends to infinity, the mixed layer depth becomes independent of the Ekman layer length scale and dependent only upon the Monin-Obukov length. Hence, very near the equator, where the Coriolis parameter f tends to zero, the mixed layer depth becomes independent of f.

As the ratio of the Ekman and Monin-Obukov length scales decreases, the Vager-Zilitinkevich and Mellor-Yamada models predict that the mixed layer depth becomes increasingly dependent upon the Coriolis parameter. For example, for values of ϕ_2 corresponding to a surface heat flux of 200 to 400 cal/cm^2-day, a wind stress of about one dyne, and a value of the Coriolis parameter of about 5×10^{-5} sec^{-1}, the dependence of the mixed layer depth upon the problem parameters is approximately given by

$$h \propto Ri_c^{1/4} \frac{u_\tau^2}{\sqrt{f\beta gQ}} . \qquad (41)$$

As the surface heat flux Q tends to zero, it appears that the mixed layer depth depends only upon the Ekman length scale

u_τ/f. This is because, as an initial condition for the problem of heat dominated deepening, the upper ocean was assumed to be unstratified. Since the ocean is usually not unstratified to great depths, it might be expected that for small values of the surface heat flux, the depth of the mixed layer would be limited by the existing stratification.

The studies of the behavior of the diffusion models for cases of wind dominated and heat dominated deepening of the mixed layer were made assuming simple, idealized initial and boundary conditions. In simulations of the actual ocean, the stratification is rarely ever linear, and the surface wind stress and heat flux usually do not remain constant for extended periods of time. However, the behavior of the mixed layer for more realistic conditions can be generalized from the specific situations studied here.

Mixing within the mixed layer can usually be classified as heat dominated, wind dominated, or convective depending upon the surface boundary conditions and the stratification. Given a specific wind stress and nonnegative surface heat flux, the mixed layer depth will tend towards a fairly stationary value where it is in quasi-equilibrium with the existing stratification and boundary conditions. This depth is limited either by the stratification in the case of wind dominated mixing or by the surface heat flux in the case of heat dominated mixing. If on the other hand, the net surface heat flux is negative, convective mixing will be expected to deepen the mixed layer at least to a depth sufficient to relieve the density instability and probably somewhat deeper due to convective penetration.

The time required for the mixed layer depth to adjust to a change in the surface wind stress or heat flux can be a few hours or as long as several days. Hence, if the time scale of changes in the surface boundary conditions is fairly short, as in the case of a diurnal heating cycle, the mixed layer depth may seldom be in equilibrium with the surface boundary conditions and the stratification. As a result, the adjustments of the mixed layer depth to short time scale fluctuations in the boundary conditions may be considerably damped.

On a seasonal time scale, the adjustment period of the mixed layer is negligible. Hence, the equilibrium mixed layer depths determined for the various cases of mixed layer deepening should be useful for estimating seasonal mean values of the depth of the mixed layer. For example, the equilibrium mixed layer depths determined for the case of heat dominated mixing might be used to estimate the mean mixed layer depth in the spring and summer when the upper ocean is being warmed by solar radiation.

There is a question of how well diffusion models such as those compared here can simulate actual mixing in the upper ocean. This question is best answered by comparison of the models with observations. Unfortunately, there have been few such comparisons, mainly because of the difficulty of obtaining observations of mixed layer deepening suitable for evaluating the mixed layer models.

Munk and Anderson (1948) compared their model with an accumulation of data from the Pacific taken during the summer at latitudes from 15°N to 50°N and found that the computed mixed layer depths were less than half the observed depths. Munk and Anderson did not determine whether the discrepancy was the fault of the observations or the model. Denman

and Miyake (1973) compared the integral model of Kraus and Turner with data taken at weather station 'Papa' in the North Pacific and obtained fairly good agreement with the observed temperature profiles. Mellor and Durbin (1975) used the same data from station 'Papa' to evaluate the Mellor-Yamada model and also obtained fairly good agreement with the observations. Since the models of Kraus and Turner and Mellor and Yamada have some very fundamental differences, it might be assumed that comparisons of the mixed layer models with observations that have been published to date have not been sufficient to distinguish one type of model as being superior to another.

However, on the basis of physical reasoning, it does seem a severe short coming of the Kraus and Turner and Denman models that they do not depend upon the earth's rotation. If the mixed layer is mixed by wave generated turbulence, then indeed the earth's rotation would probably not influence the mixed layer depth. But there is some doubt that mixing by surface waves penetrates sufficiently to cause the observed erosion of the thermocline at depths below 20 to 30 meters. The formation and deepening of the mixed layer due to the generation of turbulence by the instability of the mean shear in the wind driven surface Ekman layer is very nicely predicted by the equations of fluid motion and simple turbulence theory. And of the arguments against this theory, that the predicted mixed layers are not deep enough, that the horizontal length scale is not long enough for rotation to be important, and that Ekman layers have not been observed in the ocean, none really holds up.

Mellor and Durbin have shown that a stratified, turbulent, Ekman layer model can predict mixed layer depths in close agreement with those

that are observed. The drift of the surface current to the right of the wind stress as predicted by Ekman's theory, and the inertial oscillations of the mixed layer generated by changes in the wind stress demonstrate the influence of the earth's rotation on surface currents in the ocean. And Ekman spirals in the Great Lakes have been observed and simulated using the Ekman layer equations by Gonella (1971) and Csanady (1972). It should be expected that the detection of Ekman spirals at sea might be difficult due to the problems of making accurate current measurements and due to the complicating effects of surface waves, inertial oscillations, background currents, and temporal and spacial variations in the wind stress and stratification. Ekman spirals may be more frequently observed in the ocean as current measuring techniques improve.

Of the turbulent diffusion models discussed and compared here, the Mellor-Yamada model would seem to be the most suitable for modeling the mixed layer. The criterion of a Richardson number of about 1/4 for the stabilization of shear generated turbulence by a density stratification is a fairly general result, and it has been seen that the value of the Richardson number for which turbulent mixing is suppressed regulates to a large extent the mixed layer depths predicted by these types of models.

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